

Computation of All the Amicable Pairs Below 10^{10}

By H. J. J. te Riele

Abstract. An efficient exhaustive numerical search method for amicable pairs is described. With the aid of this method all 1427 amicable pairs with smaller member below 10^{10} have been computed, more than 800 pairs being new. This extends previous exhaustive work below 10^8 by H. Cohen. In three appendices (contained in the supplements section of this issue), various statistics are given, including an ordered list of all the gcd's of the 1427 amicable pairs below 10^{10} (which may be useful in further amicable pair research). Suggested by the numerical results, a theorem of Borho and Hoffmann for constructing APs has been extended.

1. Introduction. Let $\sigma(m)$ denote the sum of all the divisors of m , including 1 and m . An *amicable* pair (AP) is a pair of positive integers (m, n) , $m < n$, such that $\sigma(m) = \sigma(n) = m + n$. We note that m is *abundant* (since $\sigma(m) > 2m$) and that n is *deficient* (since $\sigma(n) < 2n$). The smallest AP is

$$(220, 284) = (2^2 5 \cdot 11, 2^2 71).$$

In order to check whether or not a given positive integer m is the smaller member of an amicable pair, it seems necessary, at first sight, to compute $\sigma(m)$ and $n := \sigma(m) - m$, to check whether $n > m$ (i.e., whether m is abundant), and, if so, to compute $\sigma(n)$ and compare $\sigma(m)$ with $\sigma(n)$. This involves one or two complete factorizations, in case m is deficient or abundant, respectively. However, a closer look reveals that it is often possible to find out whether a given number m is deficient (hence cannot be the smaller member of an AP) without the need to factorize it completely. Moreover, once $\sigma(m)$ and $n (= \sigma(m) - m)$ have been computed, it is often possible to discover that $\sigma(n) \neq \sigma(m)$ without the need to factorize n completely.

These considerations have guided the design of an efficient exhaustive numerical AP search algorithm, the details of which are given in Section 2. With the aid of this algorithm we have extended Cohen's exhaustive list of all 236 APs with smaller member below 10^8 [4] to all 1427 APs with smaller member below 10^{10} . Of these, 601 have been published earlier [6], [7]. The other 826 seem to be new, and are published here for the first time (9 of them have been communicated to the author already in 1983 and 1984 by Woods (2), Borho (2) and Lee (5)). Section 3 presents details of the computations together with several tables collected from this search. Moreover, a result of Borho and Hoffmann for constructing APs is extended, as was suggested by the numerical tables.

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Three appendices to this paper appear in the supplements section of this issue. These may also be obtained by writing to the author.

In Appendix I, we present the complete list of all 1427 APs with smaller member below 10^{10} ordered according to the size of the smaller members of the pairs. Appendix II displays the same list with a different ordering, viz., according to the various occurring types (defined in Section 3). Finally, Appendix III tabulates all the greatest common divisors of the 1427 APs, in increasing order, together with their frequencies of occurrence, and, for each gcd g , the rank numbers of all the APs (m, n) for which $\gcd(m, n) = g$.

2. Check Whether a Given m is the Smaller Member of an AP. Let p_i be the i th prime, $P_{i,j} := \prod_{k=i}^{i+j-1} p_k$, $Q_{i,j} := \prod_{k=i}^{i+j-1} p_k / (p_k - 1)$. We start with the following lemma which gives an upper bound for $\sigma(m)/m$.

LEMMA 2.1. *If m only has prime divisors $\geq p_i$ ($i \geq 1$) and if $m < P_{i,j+1}$ ($j \geq 1$) then $\sigma(m)/m < Q_{i,j}$.*

Proof. Since $m < P_{i,j+1} = p_i p_{i+1} \cdots p_{i+j}$, and since any prime divisor of m is $\geq p_i$, it follows that m has at most j different prime divisors $\geq p_i$ (otherwise we would have $m \geq p_i p_{i+1} \cdots p_{i+j} = P_{i,j+1}$). This implies that

$$\frac{\sigma(m)}{m} = \prod_{p^e \parallel m} \frac{p^{e+1} - 1}{p^e(p-1)} = \prod_{p^e \parallel m} \frac{p - p^{-e}}{p - 1} < \prod_{p \mid m} \frac{p}{p-1} \leq \prod_{k=i}^{i+j-1} \frac{p_k}{p_k - 1} = Q_{i,j}. \quad \square$$

In the algorithm below, this lemma is invoked very frequently. Therefore, we require a precomputed table of P - and Q -values, large enough so that the values needed can be found quickly by simple table look-ups.

Now we describe an efficient algorithm to check whether a given positive integer m belongs to an AP (m, n) with $m < n$. This algorithm is based on the observation that when, for given γ and N , we want to verify one of the relations $\sigma(N)/N > \gamma$, $= \gamma$, $< \gamma$, and when the primes $2, 3, \dots, p$ have been tried as divisors of N , it may be possible

(i) to detect, with Lemma 2.1, whether $\sigma(N)/N < \gamma$ by using the information that the *unfactored* portion of N only has prime divisors $> p$, and

(ii) to detect whether $\sigma(N)/N > \gamma$ by using the *factored* portion of N .

In this way, much unnecessary factorization time may be avoided. The price to pay for this gain lies in the time needed to consult the P - and Q -tables used in Lemma 2.1. In the algorithm, the index i_{\max} is the maximum value of i for which Lemma 2.1 is invoked. In order to restrict this table look-up time, i_{\max} should not be chosen too large. The optimal value of i_{\max} also depends on the actual implementation of the algorithm (cf. Section 3).

Algorithm to Check Whether m is the Smaller Member of an AP.

Step 1. (Find out whether m is abundant; in this step, keep $m = m_1 m_2$ where $\gcd(m_1, m_2) = 1$, m_1 is the factored and m_2 is the unfactored portion of m , $\alpha := \sigma(m_1)/m_1$; start with $m_1 := 1$, $m_2 := m$, $\alpha := 1$.)

Start factoring m by trial dividing m_2 by the primes $p_1, p_2, \dots \leq m_2^{1/2}$. In case a prime power divisor p_{i-1}^e ($e \geq 1$) of m_2 has been found, update m_1 , m_2 and α ($m_1 := m_1 p_{i-1}^e$, $m_2 := m/m_1$, $\alpha := \alpha \cdot \sigma(p_{i-1}^e)/p_{i-1}^e$). After the trial division with p_{i-1} (whether or not p_{i-1} divides m_2): if $\alpha < 2$ and $4 \leq i \leq i_{\max}$, check whether m

is possibly deficient as follows: by inspecting the P -table find the smallest value of j ($=:j^*$) such that $m_2 < P_{i,j+1}$; if $\alpha Q_{i,j^*} < 2$, then STOP (because, in that case, m is deficient: by Lemma 2.1 we have $\sigma(m_2)/m_2 < Q_{i,j^*}$ so that

$$\frac{\sigma(m)}{m} = \frac{\sigma(m_1)}{m_1} \cdot \frac{\sigma(m_2)}{m_2} = \alpha \frac{\sigma(m_2)}{m_2} < \alpha Q_{i,j^*} < 2.$$

If $\alpha \geq 2$, or $i < 4$ or $i > i_{\max}$, the deficiency check on m is left out. After the complete factorization of m (and simultaneous computation of $\sigma(m)$): if $m < \sigma(m) - m =: n$ (i.e., m is abundant), go to Step 2, otherwise STOP.

End of Step 1

Step 2. (Given m , $\sigma(m)$ and $n = \sigma(m) - m$, check whether $\sigma(n) = \sigma(m)$; during the factorization of n try to exclude those m for which $\sigma(n) \neq \sigma(m)$ as early as possible by testing whether $\sigma(n)/n \neq \beta$ where $\beta = \sigma(m)/n$; in this step, keep $n = n_1 n_2$, where $\gcd(n_1, n_2) = 1$, n_1 is the factored and n_2 the unfactored portion of n , $\alpha := \sigma(n_1)/n_1$; start with $n_1 := 1$, $n_2 := n$, $\alpha := 1$.)

Start factoring n by trial dividing n_2 by the primes $p_1, p_2, \dots \leq n_2^{1/2}$. In case a prime power divisor p_{i-1}^e ($e \geq 1$) of n_2 has been found, update n_1 , n_2 and α : if the updated α satisfies $\alpha > \beta$, then STOP (because, in that case, we have

$$\frac{\sigma(n)}{n} = \frac{\sigma(n_1)}{n_1} \frac{\sigma(n_2)}{n_2} \geq \frac{\sigma(n_1)}{n_1} = \alpha > \beta = \frac{\sigma(m)}{n},$$

so that $\sigma(n) \neq \sigma(m)$). After the trial division with p_{i-1} (whether or not p_{i-1} divides n_2): if $4 \leq i \leq i_{\max}$ check whether $\sigma(n)/n < \beta$ as follows: by inspecting the P -table find the smallest value of j ($=:j^*$) such that $n_2 < P_{i,j+1}$. If $\alpha Q_{i,j^*} < \beta$, then STOP (because, in that case, $\sigma(n)/n < \beta$: by Lemma 2.1 we have $\sigma(n_2)/n_2 < Q_{i,j^*}$ so that

$$\frac{\sigma(n)}{n} = \frac{\sigma(n_1)}{n_1} \cdot \frac{\sigma(n_2)}{n_2} = \alpha \frac{\sigma(n_2)}{n_2} < \alpha Q_{i,j^*} < \beta).$$

If $i < 4$ or $i > i_{\max}$, the check on $\sigma(n)/n < \beta$ is omitted. After the complete factorization of n (and simultaneous computation of $\sigma(n)$): check whether $\sigma(n) = \sigma(m)$. If so, (m, n) is an AP.

End of Step 2

3. Computing All the APs Below 10^{10} . In order to compute all the APs (m, n) with $m < n$ and $10^8 < m \leq 10^{10}$ (thus extending H. Cohen's computations reported in [4]), we distinguish between $m \equiv 0 \pmod{6}$ (the easy case), and $m \not\equiv 0 \pmod{6}$ (the hard case).

If $m \equiv 0 \pmod{6}$ and $n = \sigma(m) - m$ is even, then (m, n) cannot be an AP [5]. Therefore, n should be odd. In that case, we have [6] $m = 2^\mu M^2$, $n = N^2$, with $\mu \in \mathbb{N}$, M and N being odd. For all the numbers $m = 2^\mu M^2$ with $3 \mid M$ and $10^8 < m \leq 10^{10}$, we computed $n := \sigma(m) - m$ and checked whether n was a perfect square. Not a single such case was found. Computer time was about 6 CPU seconds.

For all $m \not\equiv 0 \pmod{6}$ with $10^8 < m \leq 10^{10}$ we used the algorithm of Section 2 to find all APs in this range. The optimal choice of i_{\max} for our FORTRAN-implementation on a CYBER 750 was about 75. This value was chosen to be fixed for the whole range. The speed-up factor of our program was about 15, compared with a

straightforward program which, given m , computes $\sigma(m)$ and, if $n := \sigma(m) - m > m$, computes $\sigma(n)$. A slight increase of the speed was obtained as follows. In Step 1, in case a prime (power) factor of m_2 was found and m_1 and $\sigma(m_1)$ (among others) were updated, it was checked whether both m_1 and $\sigma(m_1)$ were divisible by one of the primitive abundant numbers $20 = 2^2 \cdot 5$, $28 = 2^2 \cdot 7$, $70 = 2 \cdot 5 \cdot 7$ and $88 = 2^3 \cdot 11$. If so, the algorithm was stopped since this implied that also m and $\sigma(m)$, hence also $n = \sigma(m) - m$ were divisible by this abundant number, so that both m and n were abundant. This is impossible for an AP (m, n) .

The total time to cover the range $10^8 < m \leq 10^{10}$ was about 1000 (low priority) CPU hours, spent in the last seven months of 1984.

The total number of APs (m, n) found with $m < n$ and $10^8 < m \leq 10^{10}$ was 1191. In Appendix I (of the supplements section) all the APs with smaller member $\leq 10^{10}$ are given (including the 236 APs with smaller member $\leq 10^8$). For each pair we list the decimal representation and the prime factorization of the members, a rank number, a code (letter plus digit) referring to the discoverer, and the type of the pair (defined below). For example, pair #1427 reads as follows:

1427	9967523980	2E2.257.5.17.37.3083
R942	12890541236	2E2.257.107.117191.

Table 1 gives the meaning of the codes, and their frequencies of occurrence. Extensive information about the sources of the pairs with code L1 is given in the survey paper [6].

There are 1015 pairs with even members and 412 with odd members. The minimal and maximal values of m/n are 0.6979 and 0.999858 for the APs #567 and #1010, respectively.

Let $A(x)$ be the number of APs (m, n) with $m < n$ and $m \leq x$. From the list of APs with $m \leq 10^8$, Bratley et al. [3] concluded that for $x \leq 10^8$, $A(x)$ is approximately proportional to $x^{1/2}/\ln(x)$. In Table 2 we give, for $x = k \cdot 10^9$ ($1 \leq k \leq 10$): $A(x)$, $A(x)\ln(x)/x^{1/2}$, $A(x)(\ln(x))^2/x^{1/2}$ and $A(x)(\ln(x))^3/x^{1/2}$. From these figures we may draw the conclusion that for $x \leq 10^{10}$, $A(x)$ is approximately proportional to $x^{1/2}/(\ln(x))^3$.

TABLE 1

Status list of the first 1427 APs (m, n) , $m < n$, with $m \leq 10^{10}$

code	#APs	references and remarks
L1	508	[6]
R2	1	[9] (#1056)
W1	73	sent to the author by D. Woods on June 29, 1982 and published in [7]
R3	19	found by the author with the methods described in [8], and published in [7]
W2	1	sent in by D. Woods on Feb. 16, 1983 (#330)
R6	1	found by the author in May, 1983 (#1375)
W3	1	sent in by D. Woods on July 11, 1983 (#1050)
L2	5	sent in by E. J. Lee in July, 1984 (# #778, 860, 894, 1241, 1261)
B4	2	sent in by W. Borho on Nov. 2, 1984 (# #809, 1393)
R9	816	found by the author during the systematic search described in this paper

TABLE 2

Comparison of $A(x)$ with $x^{1/2}/(\ln(x))^i$, $i = 1, 2, 3$

$x/10^9$	$A(x)$	$A(x)\ln(x)/x^{1/2}$	$A(x)(\ln(x))^2/x^{1/2}$	$A(x)(\ln(x))^3/x^{1/2}$
1	586	0.3840	7.958	164.9
2	762	0.3649	7.815	167.4
3	898	0.3578	7.807	170.4
4	1009	0.3527	7.799	172.4
5	1100	0.3474	7.759	173.3
6	1185	0.3444	7.755	174.6
7	1256	0.3403	7.715	174.9
8	1317	0.3358	7.656	174.6
9	1377	0.3327	7.625	174.8
10	1427	0.3286	7.566	174.2

We define an AP (m, n) , $m < n$, to be a *regular amicable pair of type (i, j)* , if $(m, n) = (gM, gN)$, where $g = \gcd(m, n)$, $\gcd(g, M) = \gcd(g, N) = 1$, M and N are squarefree, and the numbers of prime factors of M and N are i and j , respectively. Other pairs are called *irregular* or *exotic*. There are 1082 regular and 345 irregular APs with smaller member $\leq 10^{10}$. It is easy to see that there are no regular pairs of type $(1, j)$, $j \geq 1$: let g be the gcd of such an AP, so that $(m, n) = (gp, gN)$ where p is a prime and $\gcd(g, p) = \gcd(g, N) = 1$. We have $m < n$, hence $p < N$. By definition, $\sigma(gp) = \sigma(gN)$, implying that $p + 1 = \sigma(N)$. Since, for any $N \in \mathbb{N}$, $\sigma(N) > N$, this implies that $p + 1 > N$, a contradiction. We note that in this argument N need not be squarefree.

In Table 3 we give the frequency distribution of the various types among the first 1082 regular APs. We note that there are relatively few regular APs of type $(i, 1)$, $i \geq 2$, and of type (i, j) with $i < j$.

In [7] the total number of known APs with smaller member $\leq 10^{10}$ was 601 (these are the APs belonging to the first four codes in Table 1). Among them were 104 irregular APs, i.e., 17.3%. Comparing this figure with the 345 irregular APs in our *complete* list of APs with smaller member $\leq 10^{10}$, i.e., 24.2%, we see that relatively many irregular APs were found in our systematic search.

In Appendix II (of the supplements section) we present lists of all the 1082 regular APs arranged according to their types, together with a list of the 345 exotic APs. This appendix may be useful for searches of APs of a special type.

The regular pairs of type $(i, 1)$, $i \geq 2$, play an important role as “mother” pairs in methods to generate new APs from given pairs. In [8] a substantial part of the new APs found there was constructed from such mother pairs. In [1], Borho and Hoffmann have partially generalized the methods from [8] by introducing the concept of a *breeder*: a breeder is a pair of positive integers (a_1, a_2) such that the equations

$$a_1 + a_2 x = \sigma(a_1) = \sigma(a_2)(x + 1)$$

TABLE 3
*Frequency distribution of the first 1082 regular APs
of type (i, j) , $i \geq 2, j \geq 1$*

$i =$	$j =$	1	2	3	4	5	row totals
2		20	67	21	4	0	112
3		16	271	280	24	0	591
4		1	78	201	63	2	345
5		0	6	18	7	3	34
column totals		37	422	520	98	5	1082

have a positive integer solution x . If x is a prime, then (a_1, a_2x) is an amicable pair. For certain breeders, called “special” breeders, Borho and Hoffmann formulate the following

THEOREM 1 [1]. *Let (a_1, a_2) be a special breeder, i.e., $a_1 = au$, $a_2 = a$, with $\gcd(a, u) = 1$. Take any factorization of $C := \sigma(u)(u + \sigma(u) - 1)$ into two different factors D_1, D_2 ($C = D_1D_2$). Then, if the numbers $s_i = D_i + \sigma(u) - 1$, for $i = 1, 2$, and also $q = u + s_1 + s_2$ are primes not dividing a , then (auq, as_1s_2) is an amicable pair.* \square

Regular APs of type $(i, 1)$, $i \geq 2$, are of the form (au, ap) , p prime, and the numbers (au, a) are special breeders which generally produce many APs with the above theorem.

In our list of 1427 APs we found a few APs, e.g., #647 and #955, which suggested that the condition $\gcd(a, u) = 1$ in Theorem 1 may be dropped. In fact, we have

THEOREM 2. *Let (au, a) be a breeder, i.e., there exists a positive integer x such that $au + ax = \sigma(au) = \sigma(a)(x + 1)$. Take any factorization of $C := (x + 1)(x + u)$ into two different factors D_1, D_2 ($C = D_1D_2$). Then, if the numbers $s_i = D_i + x$, for $i = 1, 2$, and also $q = u + s_1 + s_2$ are primes not dividing a , then (auq, as_1s_2) is an amicable pair.* \square

The proof of this theorem is left to the reader.

If $\gcd(a, u) = 1$, then $\sigma(au) = \sigma(a)\sigma(u)$, so that $x = \sigma(u) - 1$ and Theorem 2 reduces to Theorem 1. As an example, AP #955 gives the breeder (au, a) with $a = 3.5.7.19$ and $u = 7.29.47.181$. Theorem 2 yields 16 new APs with this breeder as input.

It is known [5] that most even APs have a pair sum which is $\equiv 0 \pmod{9}$. Our search proves that indeed Poulet’s pair #503: $(2^4331.19.6619, 2^4331.199.661)$ is the smallest exceptional pair. All known exceptional pairs had members $\equiv 7 \pmod{9}$ and a pair sum $\equiv 5 \pmod{9}$. In our search, we found two even APs with pair sum $\equiv 3 \pmod{9}$, viz., the (irregular) pairs:

$$\#577: 2^4 \left\{ \begin{array}{l} 19^2103.1627 \\ 3847.16763 \end{array} \right. \quad \text{and} \quad \#874: 2^219 \left\{ \begin{array}{l} 13^237.43.139 \\ 41.151.6709. \end{array} \right.$$

TABLE 4
The 17 APs among the first 1427, whose pair sum is $\not\equiv 0 \pmod{9}$

	even members	odd members
regular	# 503, type (2,2)	# 899, type (3,2)
	# 1031, type (2,2)	# 1057, type (2,2)
	# 1081, type (2,2)	# 1158, type (3,2)
irregular	# # 577, 874	# # 7, 38, 78, 113, 256, 440, 1083, 1175, 1380

TABLE 5
All (37) pairs from the first 1427 APs having the same pair sum

rank numbers	pair sum	prime decomposition of the pair sum, i.e., exponents belonging to the primes										
		2	3	5	7	11	13	17	19	23	31	37
32	35	1296000	7	4	3							
105	109	20528640	9	6	1							
137	138	37739520	10	4	1	1						
172	173	75479040	11	4	1	1						
272	276	321408000	10	4	3							1
282	286	348364800	13	5	2	1						
350	351	556839360	6	6	1	1	1					
347	355	579156480	9	5	1	2						1
373	375	638668800	12	4	2	1	1					
368	377	661893120	12	5	1	1						1
395	399	761177088	10	5	1						1	1
411	415	796340160	6	5	1	2	1					1
427	433	883872000	8	4	3		1					1
462	476	1181174400	7	5	2	2						1
486	491	1282417920	8	5	1	1				1		1
574	582	2068416000	9	5	3	1						1
626	630	2395008000	10	5	3	1	1					
653	665	2682408960	12	5	1	2	1					
695	697	3155023872	11	4		1	1	1				
717	730	3599769600	13	4	2	1						1
751	753	4049740800	10	6	2	1						1
798	807	4606156800	13	3	2	2			1			1
786	787	4716601344	13	2		1		1	1			
824	840	5094835200	10	7	2	1			1			
940	941	6824563200	9	3	2	2		1				1
926	952	6897623040	13	7	1	1	1					
997	998	7925299200	11	5	2	2		1				
1012	1019	8273664000	11	5	3	1			1			
1069	1097	10027929600	12	5	2			1				1
1124	1142	11195712000	9	3	3		1		1			1
1147	1150	11416204800	9	4	2	1	2	1				
1143	1181	12098211840	12	5	1		1	1	1			
1232	1233	13473008640	10	5	1	2		1	1			
1254	1265	14341017600	12	4	2	1		1				1
1249	1255	14478912000	9	5	3	2						1
1272	1278	15058068480	10	5	1	2		1				1
1410	1425	19926466560	14	5	1	1	1	1				

These are the first two examples of APs of the form described in [5, Theorem I, case (b)] (also cf. the remarks immediately following Table I in [5]). Table 4 gives the rank numbers of the 17 APs with smaller member $\leq 10^{10}$ whose pair sum is $\not\equiv 0 \pmod{9}$, divided into even and odd pairs, and regular and irregular pairs.

Another question, suggested by Professor C. Pomerance, is whether pairs, triples, quadruples, etc. of APs exist having the *same pair sum*. Among the first 1427 APs, we found 37 such pairs of APs, but no such triples, quadruples, etc. Table 5 gives the rank numbers of these pairs of APs, and the prime factorization of their pair sums. The pair sums only have prime divisors ≤ 37 . In 30 of the 37 cases at least one member of the pair was found during the exhaustive search described in the present paper.

In Appendix III (of the supplements section) we tabulate all the greatest common divisors of the first 1427 APs, ordered according to their size, with frequencies, and with the rank numbers of all the APs corresponding to a given gcd. This might be useful in further searches for special APs, and in searches for so-called *isotopic* APs (cf., [6, p. 83]). For example, new APs, isotopic with APs from the list of 1427 APs, are obtained by replacing the common factor $3^3 5$ in # # 882 and 1087 by $3^2 7.13$, by replacing the common factor $3^3 5^3$ in # 1205 by $3^2 5^2 31$, and by replacing the common factor $3^3 5^2 31$ in # # 717 and 1228 by $3^6 5.23.137.547.1093$, and by $3^{10} 5.23.107.3851$.

In [8], we have presented methods to find new APs from known APs. By applying these methods to the new APs among the first 1427 APs, we have found 117 new APs (with smaller member $> 10^{10}$). The new APs were found mainly from mother pairs having a relatively simple structure, like those of type $(i, 1)$, $i > 1$. They will be published in a forthcoming report [2], together with many other new amicable pairs.

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By H. J. J. te Riele

Appendix I

The first 1427 APs

L1 21 220 2E2.5.11 3D	31 600392 2E3.13.23.251 6D
2 1184 2E5.37	32 669688 2E3.97.863
L1 X 1210 2.5.11E2	32 609928 2E3.11.29.239
3 2628 2E2.5.131	32 686072 2E3.191.449
L1 22 2924 2E2.17.43	33 624184 2E3.11.41.173
4 5028 2E2.5.251	32 691256 2E3.71.1217
L1 22 5564 2E2.13.107	34 635624 2E3.11.31.233
5 6232 2E3.19.41	32 712216 2E3.127.701
L1 X 6368 2E5.199	35 643336 2E3.29.47.59
6 10744 2E3.17.79	32 652664 2E3.17.4799
L1 22 10856 2E3.23.59	36 667964 2E2.11.17.19.47
7 12285 3E3.5.7.13	32 783556 2E2.31.71.89
L1 X 14595 3.5.7.139	37 726184 2E3.17.19.281
8 17296 2E4.23.47	32 796696 2E3.53.1879
L1 21 18416 2E4.1151	38 802725 3.5E2.7.11.139
9 63020 2E2.23.5.137	L1 X 863835 3.5.7.19.433
L1 21 76084 2E2.23.827	39 879712 2E5.37.743
10 66928 2E4.47.89	L1 X 901424 2E4.53.1063
L1 22 66992 2E4.53.79	40 898216 2E3.11.59.173
11 67095 3E3.5.7.71	L1 32 980984 2E3.47.2609
L1 22 71145 3E3.5.17.31	41 947835 3E3.5.7.17.59
12 69615 3E2.7.13.5.17	L1 32 1125765 3E3.5.31.269
L1 21 87633 3E2.7.13.107	42 998105 2E3.17.41.179
13 79750 2.5E3.11.29	L1 32 1843099 2E3.23.5669
L1 X 88730 2.5.19.467	43 1677899 2.5.11.41.239
14 100485 3E2.5.7.11.29	L1 33 1699390 2.5.17.29.223
L1 32 124155 3E2.5.31.89	44 1154450 2.5E2.11.2099
15 122265 3E2.5.13.11.19	L1 22 1189150 2.5E2.17.1399
L1 21 139815 3E2.5.13.239	45 1156870 2.5.11.13.809
16 122364 2E9.239	L1 32 1292570 2.5.19.6803
L1 X 123152 2E4.13.179	46 1175265 3E2.7E2.13.251
17 141664 2E5.19.233	L1 21 1438983 3E2.7E2.13.251
L1 X 153176 2E3.41.467	47 1185376 2E5.17.2179
18 142316 2.5.7.19.107	L1 X 1286744 2E3.41.3923
L1 32 168730 2.5.47.359	48 1280565 3E2.5.13.11.199
19 171856 2E4.23.467	L1 22 1340235 3E2.5.13.29.79
L1 22 176336 2E4.103.107	49 128470 2.5.11.13.929
20 176272 2E4.23.479	L1 X 1483056 2.5E2.59.503
L1 22 188848 2E4.89.127	50 1358595 3E2.5.19.7.227
21 185364 2E3.17.29.47	L1 22 1486845 3E2.5.19.37.47
L1 32 203432 2E3.59.41	51 1392364 2E4.17.5119
22 196724 2E2.11.17.263	L1 22 1464592 2E4.239.383
L1 22 202444 2E2.11.43.107	52 1466158 2.5E2.7.59.71
23 280540 2E2.5.13E2.83	L1 X 1747930 2.5.47.3719
L1 X 365884 2E2.107.853	53 14683124 2E2.11.13.17.151
24 308624 2E2.5.13.1187	L1 43 1749212 2E2.37.55.223
L1 32 389924 2E2.43.2267	54 1511398 2.5.7.21599
25 319556 2.7.5E2.11.83	L1 23 1598470 2.5.19.47.79
L1 X 430402 2.7.71.433	55 1669310 2.5.11.17.19.47
26 356408 2E3.13.23.149	L1 42 2062570 2.5.239.863
L1 32 399592 2E3.199.251	56 1798875 3E3.5E3.13.41
27 437456 2E4.19.1439	L1 X 1870245 3E2.5.13.23.139
L1 22 455344 2E4.149.191	57 2082464 2E5.59.11E3
28 469828 2E2.7E2.2393	L1 22 2090656 2E5.79.827
L1 X 486170 2.7E2.11E2.41	58 2236570 2.5.7.89.359
29 503856 2E4.23.1367	L1 33 242903 2.5.23.59.179
L1 22 514736 2E4.53.607	59 2652728 2E3.13.23.1109
30 522405 3E2.5.13.19.47	L1 32 2941672 2E3.71.5179
L1 22 525915 3E2.5.13.29.31	60 2723792 2E4.37.43.107
	L1 32 2874064 2E4.263.683

61	2738726	2.7	11.13	29.47	70	91	7577734	2.5E2	11.1	21.599	110	
l.1	43	3077354	2.7	19.23	503	l.1	32	8439380	2.5E2	5	2379	
l.1	62	7677704	2E3	11.163	19.191	l.1	22	7677728	2E3	11.139	863	
l.1	32	298136	2E3	11.187	18.07	l.1	22	7684672	2E6	16.7	863	
l.1	63	282416	2E3	11.17	19.383	l.1	22	7684672	2E5	4.5	5669	
l.1	71	22	2947216	2E4	14.7	11.03	l.1	71	7916656	2E3	11.17	58211
l.1	64	2824550	2E2	5.1	13.1	41.263	l.1	71	7916652	2E4	11.17	58211
l.1	42	371164	2E2	.503	18.47	l.1	9	7838736	2E2	11.17	59.149	
l.1	65	3216856	2E3	11.1	23.	16.19	l.1	9	8074248	2E3	11.1	59.159
l.1	32	3712154	2E3	11.136	647	719	l.1	33	83136	2E2	11.17	79.769
l.1	70	3666550	2.5E2	31.1	13.179	l.1	9	83136	2E2	11.17	83.839	
l.1	X	3826270	2.5E2	31.1	13.179	l.1	9	8619765	3.5	11.1	71.439	
l.1	67	3766394	2E3	11.1	23.	18.71	l.1	11	8619765	3.5	11.1	23.433
l.1	32	4300316	2E3	11.1	23.	18.71	l.1	9	868666	2E2	5.2	23.83
l.1	X	4456850	2.5E2	4.67	11.51	l.1	43	868666	2E2	4.7	11.51	
l.1	68	3865264	2E4	29.5	19.139	l.1	9	8738736	2E2	4.7	11.51	
l.1	32	4066736	2E4	17.79	139	l.1	32	1089730	2E4	2.5	7.1	
l.1	69	4238984	2E3	17.	71.439	l.1	9	882070	2E0	2.5	11.1	
l.1	33	4314616	2E3	11.	23.	13.179	l.1	3	10944690	2E0	5.3	11.181
l.1	70	4246130	2.5	7	66659	l.1	9	9971885	3.62	2.	31.1	
l.1	23	4488914	2.5E2	31.	11.1	57.73	l.1	22	998575	3.62	2.	31.1
l.1	71	4239739	2.5E2	31.1	11.1549	l.1	101	9199462	2E3	23.	29.19.2087	
l.1	X	4456850	2.5E2	19.1	467	l.1	22	952984	2E3	23.	29.17.907	
l.1	102	4422765	3E2	5.1	19.1	762.107	l.1	102	952716	3E2	5.1	19.1
l.1	X	5102555	3E2	.5	19.1	139	l.1	X	10791795	3.5	5.7	11.139
l.1	72	4527170	2.7	5.	13.	17.293	l.1	93	933974	2E3	11.1	59.503
l.1	42	6159562	2.7	7.1	6.173	l.1	9	9892996	2E3	11.1	83.317	
l.1	X	6464777	2E3	11.1	6.71	71	l.1	94	99354	2E7	11.1	383
l.1	51	616274	2E3	11.1	5.71	71	l.1	21	9473856	2E7	11.1	79.73
l.1	33	5504180	2.5	7.	16.3	449	l.1	95	951919	2.5	7.1	71.727
l.1	X	563990	2.5E2	1.9	16.3	449	l.1	33	11049338	2.5	7.1	71.729
l.1	76	5170702	2E6	11.	13.2543	l.1	106	9196125	3E2	5.1	11.159	
l.1	32	5839384	2E3	11.1	13.2543	l.1	X	9196125	3E2	5.1	11.159	
l.1	77	5220910	2.7	5.4	18.182	l.1	9666596	2.5E2	5.1	19.67	667	
l.1	X	5795952	2.7	5.4	18.182	l.1	X	1092298	2.5E2	5.1	19.67	31.7
l.1	78	5319725	3.7	1.3	15.87	l.1	9	1075195	3.7	1.3	15.87	
l.1	X	5646179	3.7	1.3	15.87	l.1	43	1119395	3.7	1.3	15.87	
l.1	79	5385119	3.7	1.3	15.87	l.1	99	1057952	2.5	1.1	54.1759	
l.1	X	563990	2.5	7.	16.3	449	l.1	21	1063998	2.5	1.1	54.1759
l.1	33	581209	2.5	7.	16.3	449	l.1	12	1063998	2.5	1.1	54.1759
l.1	X	581209	2.5	7.	16.3	449	l.1	12	1063998	2.5	1.1	54.1759
l.1	80	545976	2E3	11.1	17.	29.293	l.1	X	1063998	2.5	1.1	54.1759
l.1	X	545976	2E3	11.1	17.	29.293	l.1	X	1063998	2.5	1.1	54.1759
l.1	81	576672	2E3	11.1	17.	29.293	l.1	11	1075195	3.5	11.159	
l.1	32	639978	2E3	11.1	17.	29.293	l.1	22	105231	4	7.149	
l.1	82	6579015	3E2	5.1	12.	27.227	l.1	11	1063998	2.5	1.1	54.1759
l.1	X	6308895	2.5E2	5.1	12.	27.227	l.1	X	1119312	2.5	1.1	54.1759
l.1	83	5846160	2E2	5.1	12.	27.227	l.1	X	1063998	2.5	1.1	54.1759
l.1	X	5493160	2E2	5.1	12.	27.227	l.1	X	1063998	2.5	1.1	54.1759
l.1	84	6339116	2E3	5.1	12.	27.227	l.1	114	1063998	2.5	1.1	54.1759
l.1	33	63118	2E3	5.1	12.	27.227	l.1	32	1297305	2E3	5.1	12.
l.1	85	637175	3E2	5.1	12.	27.227	l.1	115	1177496	2E3	5.1	12.
l.1	22	668902	2E3	11.1	20.99	l.1	43	1312976	2E3	11.1	20.99	
l.1	86	6952126	2E3	11.1	20.99	l.1	116	1255648	2E3	11.1	20.99	
l.1	X	741866	2E3	11.1	20.99	l.1	33	1210272	2E3	11.1	20.99	
l.1	87	6939160	2E3	11.1	20.99	l.1	117	1493555	2E3	11.1	20.99	
l.1	X	7493160	2E3	11.1	20.99	l.1	21	1282045	3E2	5.1	12.639	
l.1	88	725532	2E2	11.1	20.99	l.1	114	1063998	2.5	11.1	20.99	
l.1	X	741588	2E2	11.1	20.99	l.1	32	1224427	2E2	11.1	20.99	
l.1	89	7288930	2.1E3	11.1	23.43	67	l.1	32	1224427	2E2	11.1	20.99
l.1	X	8221588	2.1E3	11.1	23.43	67	l.1	X	1261622	2.1E3	11.1	20.99
l.1	90	7491112	2E3	11.1	23.43	103	l.1	22	2268932	2E3	11.1	23.43
l.1	X	13337336	2E3	11.1	23.43	103	l.1	33	46678184	2E3	11.1	23.43

301	19880155	IE2, 5.17.19, 23.571	9D	L1	331	20886395	302.5.7.11.17.4259	R9	391	347480155	IE3, 5.17.19.31.257
R9	191590085	IE2, 5.7.6.17.755	9D	L1	433	20883345	302.5.7.11.23.4313	R9	43	365917365	IE3, 5.11.29.1031
R9	1915957409	IE2, 5.11.19.6.263	9D	L1	332	20881302	302.5.7.11.23.4313	R9	32	348798488	IE3, 5.11.29.1031
R9	X	196617265	3.5.13.13.23.461	L1	42	346120148	262.17.179.2859	W1	32	371171432	IE3, 5.11.29.1031
L1	333	2062351948	2.5.13.19.37.7.887	L1	43	30825295	302.5.17.11.101.1889	R9	33	367752340	2.5.17.11.101.1889
L1	212	224045	382.5.9.37.7.803	R9	43	2062351948	303.31.894297	L1	32	39752340	2.5.17.11.121.1619
L1	304	19943498	262.1.1.151.2369	L1	43	315757995	182.5.19.192.7E2.379	L1	32	39752340	2.7.11.17.19.43.167
R9	X	2134177	282.1.1E2.1.19.7.2239	L1	43	30683995	182.5.19.192.7E2.379	R9	42	40252558	2.7.11.1863.3079
305	20842232	282.13.157.1.12269	9D	L1	32	20807377	252.5.13.89.33.4371	L1	32	34965985	182.5.19.192.7E2.379
L1	X	2067190368	285.6.665.199	L1	32	208678624	255.449.18143	L1	32	3042788	182.5.17.17.1643
306	20897985	382.5.11.17.6733	9D	L1	32	206775505	284.1.17.19.18691	R9	X	402082318	2.7.11.17.19.1643
L1	X	20619265	385.5.13.73.17.179	L1	32	20568144	284.1.17.19.599	R9	36	3061885	302.5.17.19.887
307	20165952	284.9.49.1.883	9D	L1	32	206494865	323.5.11.23.7523	L1	32	36875324	284.1.22.1.1799
L1	33	203310448	284.6.7.89.2131	L1	32	206717008	323.5.11.33.3343	L1	32	36875324	284.1.22.1.1799
308	20305622	27.1.9.11.22.2933	9D	L1	32	208804464	323.5.11.33.3343	L1	32	36875324	284.1.22.1.1799
L1	X	2106627578	2.7.1.9.11.31.41.89	L1	32	208894834	323.5.11.33.3343	L1	32	36875324	284.1.22.1.1799
309	203972715	383.5.13.17.47.61	9D	L1	32	209221192	283.5.13.65.3197	L1	32	36875324	284.1.22.1.1799
L1	X	20729525	383.5.2.31.47.61	L1	32	206671608	283.5.13.593.2953	R9	X	36720778	283.1.17.19.1661
310	204134385	3.5.7.38.6.689	9D	L1	32	204646430	283.5.17.13.47.2549	R9	42	414480872	283.1.17.19.1661
L1	42	24461935	3.5.7.38.6.689	L1	32	20441170	283.5.17.35.4159	R9	43	42840495	283.1.17.19.1479
311	204759094	283.11.13.349	9D	L1	32	205192208	284.1.13.1.25.501	R9	33	36691335	304.5.17.19.179
L1	22	226616096	283.7.79.49.797	L1	32	205429808	284.1.13.1.25.501	R9	32	31844008	284.1.13.1.25.501
312	205434365	382.7.82.13.43.167	9D	L1	32	207227335	302.5.11.17.11.50.1637	R9	32	31414555	303.5.17.19.22831
R9	X	21846628	382.7.82.13.43.4551	R9	X	20948334	2.7.13.13.59.1091	R9	33	305611246	283.1.17.17.60.151
313	208891628	382.11.12.29.547	9D	L1	32	20808995	2.5.16.1.41.31.319	R9	39	30561598	2.7.11.45.55661
R9	51	25559933	262.31.167.1229	R9	X	20554450	2.5.16.1.41.31.319	R9	X	3095315158	2.7.11.45.55661
314	209921128	283.23.29.4.9.91	9D	L1	32	20508872	283.5.16.2.47.2549	R9	X	30911686	283.1.17.1.81.263
R9	X	224452277	283.37.33.19.919	L1	32	20465248	283.5.16.5.61.67.	R9	32	30839936	283.5.17.1.81.263
315	20990704	283.29.7.9.13.13	9D	L1	32	204108440	282.5.16.7.97701	R9	32	318404235	283.5.17.1.81.263
R9	X	216616096	283.19.83.16631	L1	32	205148738	282.5.16.7.97701	R9	34	30881786	283.5.17.1.81.263
316	211319745	383.5.13.1.13.167	9D	L1	32	205619278	282.5.16.7.9859	R9	32	30988165	283.5.17.1.81.263
R9	55	234887994	3.7.11.17.31.41.47	L1	32	207227335	302.5.11.17.11.50.1637	R9	32	31339524	303.5.17.1.81.263
317	214448488	283.11.17.19.41.2027	9D	L1	32	20883345	302.5.11.17.11.50.1637	R9	33	31919155	303.5.17.1.81.263
L1	42	245509191	283.11.1667979	L1	32	208813444	283.5.11.17.12.17.179	R9	34	31920958	303.5.17.1.81.263
318	216189216	283.19.52.28661	9D	L1	32	20846864	283.5.11.17.12.17.179	R9	34	31920958	303.5.17.1.81.263
L1	33	217872184	283.1.7.6.69.2441	R9	34	20251401	283.5.16.1.23.337	R9	34	31920958	303.5.17.1.81.263
319	219159175	383.5.1.1.29.65.59.63	9D	L1	32	2083310	283.5.16.1.23.337	R9	34	31920958	303.5.17.1.81.263
R9	X	219159175	383.5.1.1.29.65.59.63	L1	32	2083310	283.5.16.1.23.337	R9	34	31920958	303.5.17.1.81.263
320	223787994	383.5.1.1.29.65.59.63	9D	L1	32	20883345	302.5.11.17.11.50.1637	R9	35	31920958	303.5.17.1.81.263
L1	32	223787994	383.5.1.1.29.65.59.63	R9	X	214448488	283.5.1.1.29.65.59.63	R9	X	31920958	303.5.17.1.81.263
321	234887994	382.7.1.1.5.1.1583	9D	L1	32	208813444	283.5.1.1.29.65.59.63	R9	X	31920958	303.5.17.1.81.263
L1	32	237377269	382.1.1.83.4.751	R9	X	210207988	283.5.1.1.29.65.59.63	R9	X	31920958	303.5.17.1.81.263
322	226564032	283.4.6.65599	9D	L1	32	2083310	283.5.1.1.29.65.59.63	R9	X	31920958	303.5.17.1.81.263
L1	23	23165658	283.5.5.39.59.63.683	L1	32	2084136	283.5.1.1.29.65.59.63	R9	32	319881156	303.5.17.1.81.263
323	227134340	283.5.1.1.29.65.59.63	9D	L1	32	2083310	283.5.1.1.29.65.59.63	R9	32	319881156	303.5.17.1.81.263
R9	52	30251674	282.2.11.7.6.3743	L1	43	23328884	283.5.1.1.29.65.59.63	R9	32	319881156	303.5.17.1.81.263
324	231111392	285.7.1.10.173.383	9D	L1	32	2083345	283.5.1.1.29.65.59.63	R9	32	319881156	303.5.17.1.81.263
L1	32	2312234388	285.7.1.10.173.383	R9	X	214448488	283.5.1.1.29.65.59.63	R9	32	319881156	303.5.17.1.81.263
325	238162815	383.5.1.1.23.19.367	9D	L1	32	208813444	283.5.1.1.29.65.59.63	R9	32	319881156	303.5.17.1.81.263
L1	32	237377269	383.5.1.1.23.19.367	R9	X	210207988	283.5.1.1.29.65.59.63	R9	32	319881156	303.5.17.1.81.263
326	238338966	284.4.41.47.7727	9D	L1	32	2083310	283.5.1.1.29.65.59.63	R9	32	319881156	303.5.17.1.81.263
R9	X	234739224	284.4.97.17.1151	R9	X	227390815	283.5.1.1.29.65.59.63	R9	X	319881156	303.5.17.1.81.263
327	242126704	284.4.23.47.13999	9D	L1	32	207205032	283.5.1.1.29.65.59.63	R9	32	319881156	303.5.17.1.81.263
L1	32	257126416	284.4.23.47.13999	R9	X	227390815	283.5.1.1.29.65.59.63	R9	X	319881156	303.5.17.1.81.263
328	24228239	2.5.11.19.11.15877	9D	L1	33	23273350	2.5E.13.11.10799	R9	33	346875186	303.5.1.1.29.65.59.63
L1	33	238110030	2.5.11.19.11.24919	R9	X	227390815	2.5E.13.11.10799	R9	X	346875186	303.5.1.1.29.65.59.63
329	245184815	382.5.1.1.29.7.41039	9D	L1	32	209167945	382.5.1.1.29.7.41039	R9	33	346875186	303.5.1.1.29.65.59.63
W1	23	266369188	382.5.1.1.29.7.41039	R9	X	227390815	382.5.1.1.29.7.41039	R9	X	346875186	303.5.1.1.29.65.59.63
330	248558488	283.1322.183919	9D	L1	32	209194224	283.1.1.29.8.113.191	R9	33	346875186	303.5.1.1.29.65.59.63
W2	X	256261912	283.3743.26129	R9	X	227390815	283.1.1.29.8.113.191	R9	X	346875186	303.5.1.1.29.65.59.63

1381	9628724795	3E2-7	13-5	31-83-857	1411	9549621568	267-37-101-19963
R9 43	11111616919	BE-7	13	71-23-727	R9 12	10182996752	264-9043-703169
1382	9654431084	2B5-67	383-110-7	W1 13	9581473976	2E3-19-39-106349-67	
L1 32	9088633464	2B5-71	3999887	W1 13	9619810024	2E3-13-151-59-769	
1383	9670349152	2B5-53	83-644-39	W1 13	9616775744	2E6-83-317-57-711	
R9 32	9343487168	2B5-971	1300719	W1 13	9667754	2E6-2351-64871	
1384	907291470	2B5-782	5-23-43	1414	97038910030	2-7-17-5-47-173501	
R9 X	1161343436	2-7-41	113-24	1415	9723534388	2-7-17-1223-40223	
1385	9883122475	3-582	7-19-139	1415	9723534388	2-7-17-1223-40223	
R9 33	9115709925	3-582	7-19-139	1415	9723534388	2-7-17-1223-40223	
1386	988637043	5-53-11	83-89-171	1416	9723534388	2-7-17-1223-40223	
R9 43	9687155710	2-5-53-17	719-1483	1417	10015996666	2E4-12959-50231	
1387	9897736655	3-5-782-19	83-167	1417	9818865683	2E3-132-29-179-1399	
R9 32	94352736795	3-5-782-19	83-167	R9 9	10033693432	2E3-89-1649-14439	
1388	9108963376	2B5-79	113-3159	1418	9834708285	3E2-5-111-19-139-723	
R9 33	920412224	2B5-179	227-7879	1419	984118915	3E2-5-111-19-139-723	
1389	913652125	3E3-582	13-17-73	1419	9836611130	2-5-11-17-73-139	
R9 43	10287235575	3E3-582	59-97	1419	9836611130	2-5-11-17-73-139	
1390	91530866985	3E2-5-13-11	377-1039	1420	984469775	3E2-5-11-29-971	
R9 X	10027159055	3E2-5-13-11	3789-937	1420	984469775	3E2-5-11-29-971	
1391	91591650242	3E5-53-241	22409	1421	984525894	2E3-31-433-398	
L1 32	929042216	2B5-53-241	22409	1421	984525894	2E3-31-433-398	
1392	9166990225	3-582-19	7-17-89-667	1422	984525894	2E3-31-433-398	
R9 44	1038866175	3-582-19	31-47-99-71	1422	984525894	2E3-31-433-398	
1393	9173010856	2B5-13-11	8889-1051	1423	984655681	3E2-5-7-11-123-7919	
B4 33	915337744	2B5-13-11	201-21987	1423	984655681	3E2-5-7-11-123-7919	
R9 34	922801010	2-5-37	16-24623	1424	984881804	2E4-29-47-169-4157	
1394	10294416906	2-5-37	13-19-2749	1424	984881804	2E4-29-47-169-4157	
R9 X	10294416906	2-5-37	13-19-2749	1425	984881804	2E4-29-47-169-4157	
1395	922519116	2B5-83-13-151	23063	1425	984881804	2E4-29-47-169-4157	
L1 32	929042216	2B5-83-13-151	23063	1425	984881804	2E4-29-47-169-4157	
1396	9251688032	2B5-67-179	24197	1426	98495519	2E1-9-113-659-883	
R9 33	93384779298	2B5-101-122-2351	1397	1426	98495519	2E1-9-113-659-883	
L1 22	9357737715	3-5-7-11-37	23099	1427	98495519	2E1-9-113-659-883	
1398	9626366485	3-5-7-11-37	23099	R9 42	128890541236	2E2-257-5-1-7-31-3083	
R9 X	97366919	2-5-11-5-43	313-287				
1399	97271316500	2E2-583-53-271	6689				
R9 X	11451433932	BE2-41-101-633	883				
1400	92886441650	2-582-11-59-419-683					
R9 44	93496422158	2-582-53-113-14-9-223					
1401	9394661770	2-5-13-29-5-181-257					
R9 42	9374745678	2-5-13-29-5-181-257					
1402	9393633290	2-5-13-29-113-2139					
R9 33	93188521	2-5-13-59-339-3457					
1403	939744630	2-11-5-43-313-287					
R9 44	94577161098	2-11-5-43-17-523-6987					
1404	944411064	2B5-13-159-659-2311					
R9 44	9481776936	2B3-67-79-199-2311					
1405	9357224877	3E2-783-13-1182-41-47					
R9 X	10162433523	3E5-783-13-83-113					
1406	10374110636	3E2-11-19-53-83-2549					
R9 54	10054113364	3E2-13-7-129-1299					
1407	9498639824	3E3-13-1-59-149-251					
R9 53	1059069116	3E3-167-129-1299					
1408	94495312	2B3-19-71-139-6299					
R9 44	9602145688	2B3-29-59-179-3199					
1409	9498622048	2B5-61-486199					
L1 X	930044952	2B9-4079-4549					
1410	95399875	3E2-5-11-23-31-41-659					
R9 43	10390515795	3E2-5-11-127-197-839					

Appendix II

The first 1427 APs ordered according
to the various occurring types

AMICABLE PAIRS OF TYPE (2,1):

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1 220 2E2.5.11
L1 21 284 2B2.71
8 17294 2B4..23.47
L1 21 18416 2B4..1151
9 63026 2B2..23.5..137
L1 21 76084 2B2..23..827
12 69615 3B2..7..13..5..17
L1 21 87633 3B2..7..13..107
15 122265 3E2..7E2..13..5..41
L1 21 139815 3E2..5..13..239
46 1175265 3E2..7E2..13..5..41
L1 21 1438983 3E2..7E2..13..251
104 9363584 2E7..191..383
L1 21 9437056 2E7..73727
117 11498355 3B4..5..11..29..89
L1 21 12024045 3B4..5..11..2699
162 31536855 3B2..5..7..53..1889
L1 21 32148587 3B2..5..7..102059
291 175032884 2E2..13..17..389..509
L1 21 175826716 2E2..13..17..198899
297 183408615 3E2..5..13..19..29..569
L1 21 190055385 3E2..5..13..19..17699
303 196421715 3E2..5..19..37..7..887
L1 21 224703495 3E2..5..19..37..7103
468 536637465 3E2..7E2..13..97..5..193
L1 21 646745463 3E2..7E2..13..97..1163
629 1191953763 3E2..7E2..11..13..41..461
L1 21 1223611389 3E2..7E2..11..13..19403
648 1225052829 3E4..7..11..29..13..521
L1 21 1321639811 3E4..7..11..29..7307
792 21726492116 2E8..257..33023
L1 21 2181168894 2E8..8520191
888 2935281375 3E3..5E3..13..149..449
L1 21 2961518625 3E3..5E3..13..67499
1030 4149106335 3E4..5..11E3..43..179
L1 21 4266776545 3E4..5..11E3..7919
1191 60665249175 3E2..5E2..13..31..149..449
L1 21 6128471825 3E2..5E2..13..31..67499
1219 63708495978 2..7E2..19..23..11..13523
L1 21 6956103062 2..7E2..19..23..162287

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TOTAL NUMBER: 20

AMICABLE PAIRS OF TYPE (3,1):

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86 6955216 2E4..19..137..167
L1 31 7418864 2B4..463679
151 23358248 2E3..37..23..47..73
L1 31 25233112 2E3..37..85247
164 32205616 2E4..17..167..709
L1 31 34352624 2E4..2147039
194 52695376 2E4..17..151..1283
L1 31 56208368 2E4..3513023
270 147366765 3E2..7E2..13..5..53..97
L1 31 182028483 3E2..7E2..13..31751
312 205843365 3E2..7E2..13..5..43..167
L1 31 254264283 3E2..7E2..13..44351
394 34726321 2E4..17..137..9319
L1 31 370414064 2E4..23150879
446 492275992 2E3..131..13..23..1571
R9 31 55354416 2E3..131..528191
644 1254255550 2..582..23..19..137..419
R9 31 1333078850 2..582..23..1159199
661 1309651310 2..5..11..29..571..719
R9 31 1359071890 2..5..11..12355199
753 1957374968 2E3..31..17..107..4339
L1 31 2692365832 2E3..31..8436959
782 2115211995 3E3..5..13..17..31..2287
R9 31 2312891685 3E3..5..13..131..7887
979 3693013664 2E5..41..131..21487
L1 31 3812143072 2E5..119129471
1000 3986534090 2..5..929..7..11..5573
L1 31 4971106870 2..5..929..535103
1228 6562770525 3E3..5E2..31..17..19..971
R9 31 7322055075 3E3..5E2..31..349919
1300 7696871576 2E3..19..53..127..7523
R9 31 7904894824 2E3..19..52005887

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TOTAL NUMBER: 16

AMICABLE PAIRS OF TYPE (4,1):

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779 2099442345 3..5..7..11..13..37..3779
L1 41 2533609495 3..5..7..24131519

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TOTAL NUMBER: 1

AMICABLE PAIRS OF TYPE (2, 2);
AMICABLE PAIRS OF TYPE (3, 2);
AMICABLE PAIRS OF TYPE (4, 2);
TOTAL NUMBER: 67

L1. 22. 29224. 282. 1.131	264. 66395130. 2.5. 31. 7. 30689	1098. 4890088652. 2E2. 11. 1.109. 13. 76479
L1. 22. 29224. 282. 1.17.43	L1. 22. 743249190. 2.5. 31. 59. 4991	L1. 22. 5259164168. 2E2. 11. 1.109. 839. 1367
L1. 22. 5664. 282. 1.251	L1. 22. 943722506. 2.5. 52. 23. 137. 599	L1. 183. 59831596512. 2E5. 79. 227. 10427
L1. 22. 5664. 282. 1.13.107	L1. 22. 5020. 2.5. 20. 22. 111	L1. 22. 59994468848. 2E5. 79. 631. 3161
L1. 22. 10856. 2E3. 17.107	L1. 22. 9768994. 2E4. 67. 221. 101	L1. 199. 614353695. 3E4. 5. 11. 17. 68619
L1. 22. 10856. 2E3. 23.-59	L1. 22. 10744. 2E3. 17.107	L1. 22. 6414291585. 3E4. 5. 11. 71. 17159
L1. 22. 10856. 2E3. 47.-59	L1. 22. 9768994. 2E4. 67. 221. 101	L1. 22. 6478496595. 3E2. 5. 11. 19. 23. 19379
L1. 22. 66992. 2E4. 53.-79	L1. 22. 9768994. 2E4. 67. 221. 101	L1. 22. 6592496595. 3E2. 5. 11. 19. 37. 12329
L1. 22. 66992. 2E4. 53.-79	L1. 22. 6931045. 2E4. 5. 1. 4. 59	L1. 22. 66992. 2E2. 11. 382. 1.187
L1. 22. 66992. 2E4. 53.-79	L1. 22. 1106132995. 3E4. 5. 11. 59. 419	L1. 22. 7093163925. 3E2. 5. 1.382. 311. 399
L1. 22. 67095. 3E3. 5.-7.71	L1. 22. 13148395. 3E2. 5. 1. 2. 5910	L1. 22. 7074666624. 2E9. 34. 14591
L1. 22. 71145. 3E3. 5.-17.31	L1. 22. 132651895. 3E2. 5. 1. 2. 1889	L1. 22. 707675934. 2E9. 34. 167. 14011
L1. 19. 171866. 2E4. 23.-467	L1. 22. 132651895. 3E2. 5. 1. 2. 1889	L1. 307. 925772215. 3. 5. 7. 1. 247. 2. 20699
L1. 22. 176316. 2E4. 103. 1.107	L1. 22. 14953414. 3. 2. 7. 11. 13. 23. 559	L1. 32. 6659988. 2E3. 11. 23. 251
L1. 22. 176316. 2E4. 23.-467	L1. 22. 14953414. 3. 2. 7. 11. 13. 23. 559	L1. 32. 6659988. 2E3. 11. 23. 251
L1. 22. 186622. 2E4. 23.479	L1. 22. 1562671087. 3E3. 5. 11. 23. 7197	L1. 418. 9349789825. 3E2. 5. 11. 19. 39. 723
L1. 22. 186622. 2E4. 23.479	L1. 22. 1562671087. 3E3. 5. 11. 23. 7197	L1. 22. 9894118915. 3E2. 5. 11. 19. 39. 1647
L1. 22. 186622. 2E4. 59.122	L1. 22. 16311088. 3E3. 5. 11. 1. 3343	L1. 22. 9894118915. 3E2. 5. 11. 19. 39. 1647
L1. 22. 186724. 2E2. 11. 1.17. 263	L1. 341. 2651192208. 2E2. 131. 23. 5501	L1. 33. 624184. 2E2. 11. 41. 173
L1. 27. 457456. 2E4. 19.44.107	L1. 341. 2651192208. 2E2. 131. 23. 5501	L1. 32. 6241856. 2E3. 11. 1217
L1. 22. 505394. 2E4. 149.1.192	L1. 369. 105361984. 2E4. 331. 199. 4987	L1. 34. 633524. 2E3. 11. 323
L1. 29. 503895. 3E2. 53.6097	L1. 418. 503856064. 3E4. 337. 179. 5623	L1. 32. 6522116. 2E3. 11. 761
L1. 21. 524495. 3E2. 53.6097	L1. 418. 503856064. 3E4. 337. 179. 5623	L1. 35. 633336. 2E3. 11. 47. 59
L1. 38. 5024495. 3E2. 53.6097	L1. 418. 503856064. 3E4. 337. 179. 5623	L1. 32. 652664. 2E3. 11. 47. 59
L1. 495. 65101665. 3E2. 5. 13. 19.4217	L1. 418. 503856064. 3E4. 337. 179. 5623	L1. 37. 726104. 2E3. 11. 47. 59
L1. 22. 625915. 3E2. 5. 13. 29.31	L1. 418. 503856064. 3E4. 337. 179. 5623	L1. 32. 652672. 2E3. 11. 47. 59
L1. 44. 115450. 2. 582. 11. 1099	L1. 503. 663630256. 3E4. 331. 19. 6619	L1. 32. 666996. 2E3. 11. 47. 59
L1. 22. 108915. 2E2. 17.1.399	L1. 522. 696630544. 3E4. 331. 199. 661	L1. 32. 698984. 2E3. 11. 47. 59
L1. 48. 120956. 3E2. 5. 13. 11. 199	L1. 520. 749388964. 3E8. 383. 763	L1. 32. 7041672. 2E3. 71. 5179
L1. 21. 1340723. 3E2. 5. 13. 29.79	L1. 520. 750553992. 3E8. 1167. 1871	L1. 41. 947835. 3E3. 5. 7. 17. 59
L1. 50. 135895. 3E2. 5. 19.7.227	L1. 528. 766292835. 5. 7. 11. 53. 1319	L1. 32. 7125765. 3E3. 5. 31. 269
L1. 21. 1392045. 3E2. 5. 19.37.47	L1. 522. 766512285. 5. 7. 11. 709. 163	L1. 42. 998164. 2E3. 17. 41. 179
L1. 51. 1392045. 3E2. 5. 19.37.47	L1. 522. 766512285. 5. 7. 11. 709. 163	L1. 32. 1104096. 2E3. 22. 5669
L1. 22. 1464949. 2E4. 17.5.11.9	L1. 520. 770880945. 3E2. 7. 13. 41. 5.491	L1. 45. 11156870. 2E3. 22. 11.3. 899
L1. 22. 1464949. 2E4. 17.5.11.9	L1. 522. 9140559. 3E2. 7. 13. 41. 163. 167	L1. 32. 12957728. 2E3. 17. 18. 6003
L1. 57. 2682464. 255.59.1.1033	L1. 522. 9140559. 3E2. 7. 13. 41. 163. 167	L1. 32. 12957728. 2E3. 17. 18. 6003
L1. 21. 2890565. 255.59.1.1033	L1. 56. 90233744. 2E8. 383. 9.283	L1. 32. 13. 23. 1109
L1. 63. 2890565. 255.59.1.1033	L1. 56. 90370895. 2E8. 1151. 3067	L1. 32. 2941672. 2E3. 71. 5179
L1. 21. 2894716. 284. 17.1.1033	L1. 641. 1237888448. 2E6. 73. 26959	L1. 40. 2723704. 2E3. 31. 43. 107
L1. 72. 7654672. 285. 1.167. 1.103	L1. 641. 127405632. 2E6. 73. 49. 48487	L1. 32. 2847064. 2E4. 263. 683
L1. 85. 6317175. 3E2. 5. 22. 5E7. 1.1049	L1. 689. 15522255. 3E3. 152. 1.23. 5779	L1. 62. 2739704. 2E3. 11. 163. 191
L1. 21. 6680025. 3E2. 5E2. 11. 6699	L1. 722. 15700735. 3E3. 152. 1.23. 5779	L1. 32. 298116. 2E3. 31. 11887
L1. 97. 6939310. 2. 5. 13. 23. 239	L1. 722. 15700735. 3E3. 152. 1.23. 5779	L1. 65. 3272686. 2E3. 11. 23. 1619
L1. 22. 7158110. 2. 5. 13. 53. 1639	L1. 722. 159265161. 3E2. 7. 11. 1. 33. 152	L1. 32. 32725486. 2E3. 6. 67. 719
L1. 22. 7158110. 2. 5. 13. 53. 1639	L1. 722. 159265161. 3E2. 7. 11. 1. 33. 333	L1. 67. 3786994. 2E3. 11. 23. 1109
L1. 111. 1057550. 2. 582. 31. 19.3693	L1. 788. 1666618045. 3E4. 5. 11. 2. 71. 419	L1. 32. 3941672. 2E3. 467. 1151
L1. 21. 1057550. 2. 582. 31. 19.3693	L1. 788. 167140345. 3E4. 5. 11. 2. 71. 419	L1. 32. 40360116. 2E3. 467. 1151
L1. 125. 13921528. 283. 1.19.1.167	L1. 808. 170855. 3E2. 5. 31. 1. 929	L1. 32. 46867316. 2E4. 26. 59. 139
L1. 21. 13986672. 283. 1.19.1.167	L1. 808. 170855. 3E2. 5. 31. 1. 929	L1. 22. 46867316. 2E4. 26. 59. 139
L1. 101. 919996. 283. 1.19.2897	L1. 81. 176252896. 2E7. 38. 9.4783	L1. 45. 5117073. 2E3. 11. 21. 23. 543
L1. 22. 9392394. 283. 1.19.2897	L1. 81. 176252896. 2E7. 38. 9.4783	L1. 32. 56349688. 2E3. 303. 1.19.287
L1. 109. 10254970. 2. 5. 11. 53. 1759	L1. 829. 1786436295. 3. 5. 7. 11. 293. 5279	L1. 51. 572692. 2E3. 11. 31. 3699
L1. 22. 10273670. 2. 5. 11. 53. 1759	L1. 829. 1786436295. 3. 5. 7. 11. 293. 5279	L1. 32. 63165918. 2E3. 11. 31. 3699
L1. 111. 1057550. 2. 582. 31. 19.3693	L1. 755. 191747445. 3E4. 5. 11. 2. 59. 683	L1. 51. 572692. 2E3. 11. 31. 3699
L1. 21. 1057550. 2. 582. 31. 19.3693	L1. 755. 191747445. 3E4. 5. 11. 2. 59. 683	L1. 32. 63165918. 2E3. 11. 31. 3699
L1. 21. 1085650. 2. 582. 31. 19.3693	L1. 787. 2152573695. 3E2. 7. 13. 37. 5. 14.287	L1. 51. 572692. 2E3. 11. 31. 3699
L1. 21. 1085650. 2. 582. 31. 19.3693	L1. 787. 2152573695. 3E2. 7. 13. 37. 5. 14.287	L1. 32. 63165918. 2E3. 11. 31. 3699
L1. 125. 13921528. 283. 1.19.1.167	L1. 994. 3865936625. 3E2. 5. 11. 29. 43. 6263	L1. 51. 572692. 2E3. 11. 31. 3699
L1. 21. 13986672. 283. 1.19.1.167	L1. 994. 3865936625. 3E2. 5. 11. 29. 43. 6263	L1. 32. 63165918. 2E3. 11. 31. 3699
L1. 130. 15004264. 285. 1.17.1.1671	L1. 1031. 41052322. 2E4. 43. 61. 98999	L1. 51. 572692. 2E3. 11. 31. 3699
L1. 22. 15333030. 283. 1.17.1.1671	L1. 12. 4213181968. 2E4. 43. 65. 7129	L1. 32. 63165918. 2E3. 11. 31. 3699
L1. 139. 1790064. 285. 1.17.1.1671	L1. 1032. 43779919. 2E5. 293. 5367	L1. 51. 572692. 2E3. 11. 31. 3699
L1. 22. 16810756. 285. 79.7127	L1. 12. 43980617. 2E5. 293. 5367	L1. 32. 63165918. 2E3. 11. 31. 3699
L1. 149. 22501145. 383. 5. 11. 23. 659	L1. 1057. 449428615. 3. 5E2. 7. 19. 167. 2659	L1. 51. 572692. 2E3. 11. 31. 3699
L1. 21. 22. 2311055. 383. 5. 11. 23. 659	L1. 12. 443667055. 3. 5E2. 7. 19. 229. 3861	L1. 32. 63165918. 2E3. 11. 31. 3699
L1. 197. 5685872. 286. 79.1.1087	L1. 1081. 4796736644. 2E4. 43. 67. 100059	L1. 51. 572692. 2E3. 11. 31. 3699
L1. 22. 56598208. 286. 383.2309	L1. 12. 4855069456. 2E4. 43. 37. 1.18919	L1. 32. 63165918. 2E3. 11. 31. 3699

AMICABLE PAIRS OF TYPE (4, 3):		TOTAL NUMBER: 280
1223	6468278688 2E5, 6.1.199, 16631	R9 33 868821632 2E5, 7.1, 79, 48239
R9 33	652639712 2E5, 3.19, 3.59, 4891	R9 33 887069632 2E5, 4.0, 5.59, 1151
1225	650617325 3.5E2, 7.1, 59, 1217	R1 43 986312575 3.5E2, 7.19, 119, 6551
R9 33	652991525 3.5E2, 7.19, 521, 1229	R1 43 986312575 3.5E2, 7.29, 41, 122739
1226	6682859000 3E2, 7.13, 23, 41, 509	R1 33 9115709922 2E5, 11, 113, 31859
R9 33	679819939 3E2, 7.13, 17, 39, 67, 251	R1 33 92451224 2E5, 17, 19, 227, 3059
1228	674313936 2E4, 29, 47, 309167	R1 33 91391256 2E3, 13, 8689, 1051
R9 33	679819936 2E4, 29, 47, 309167	R1 43 3677354 2.7, 19, 23, 583
1241	676553698 2E4, 151, 431, 6779	R4 33 93331272 2.11, 5.23, 43, 67
R9 33	686866832 2E4, 371, 439, 2089, 6983	L1 43 8788952 2.11, 5.23, 43, 67
1242	6771889268 2E5, 7.1, 151, 19739	L1 43 8215958 2.11, 7.19, 211
R9 33	683291972 2E5, 11, 33, 565, 359	L1 43 8666860 2E5, 5.23, 83, 227
1247	6845165390 2E5, 11, 33, 139, 8457	L1 43 92168802 2E2, 41, 71, 797
R9 33	6950060699 2E5, 11, 33, 139, 8457	L1 43 106383928 2E5, 11, 19, 41, 103
1253	692176572 2E3, 11, 79, 996561	L1 43 108416908 2E3, 11, 19, 113, 71, 1113
R9 33	692176572 2E3, 11, 79, 996561	L1 43 127821352 2E3, 11, 19, 113, 71, 1113
1260	7096666610 2.5, 11, 53, 719, 1693	L1 43 12812076 2E2, 31, 47, 2267
R9 33	7126808910 2.5, 11, 53, 269, 2903	L1 43 14242230 2.7, 5.13, 83, 191
1261	7107900496 2E4, 191, 239, 244993	L1 43 1668988 2.7, 31, 71, 587
R9 33	7144382384 2E4, 191, 281, 8669	L1 43 18299715 2E2, 5.1, 11, 59, 89
1262	7122888110 2.5, 11, 19, 103, 2149	L1 43 1844267930 2.5, 11, 197, 571, 1259
R9 33	7303477920 2.5, 11, 19, 1039, 4427	R9 33 992985128 2E3, 11, 19, 103, 3229
1263	7341633048 2E3, 53, 83, 210719, 83	R9 33 994662872 2E3, 11, 19, 113, 659, 883
R9 33	7341633048 2E3, 53, 83, 210719, 83	L1 43 1179315 2E2, 5.1, 31, 79, 107
1274	733394892 2E4, 1, 39, 77779	L1 43 1179315 2E2, 5.1, 31, 79, 107
R9 33	7558940988 2E4, 419, 1049, 1163	L1 43 1179315 2E2, 5.1, 31, 79, 107
1276	733566710 2.5, 11, 83, 42293	L1 43 12121076 2E2, 31, 47, 2267
R9 33	8011980010 2.5, 11, 83, 291, 953	L1 43 14242230 2.7, 5.13, 83, 191
1278	7357708965 2E3, 5, 13, 17, 103, 2393	L1 43 1668988 2.7, 31, 71, 587
R9 33	736367510 2E3, 5, 13, 83, 227, 233	L1 43 18299715 2E2, 5.1, 11, 59, 89
1279	7399179525 2.5, 11, 19, 127, 3187	L1 43 1844267930 2.5, 11, 197, 571, 1259
R9 33	7399179525 2.5, 11, 19, 127, 3187	R9 33 992985128 2E3, 11, 19, 103, 3229
1282	7606666387 3.5E2, 7.17, 349, 419	R9 33 994662872 2E3, 11, 19, 113, 659, 883
R9 33	7606666387 3.5E2, 7.17, 349, 419	L1 43 1179315 2E2, 5.1, 31, 79, 107
1283	769668382 2E5, 23, 239, 2099	L1 43 12121076 2E2, 31, 47, 2267
R9 33	769668382 2E5, 23, 239, 2099	L1 43 14242230 2.7, 5.13, 83, 191
1290	771221165 2E4, 7, 11, 13, 59, 1583	L1 43 1668988 2.7, 31, 71, 587
R9 33	7893698893 2E4, 7, 11, 13, 19, 307	L1 43 18299715 2E2, 5.1, 11, 59, 89
1291	792509717 2E4, 7, 11, 13, 19, 307	L1 43 1844267930 2.5, 11, 197, 571, 1259
R9 33	792509717 2E4, 7, 11, 13, 19, 307	R9 33 992985128 2E3, 11, 19, 103, 3229
1292	7939179525 2.5, 11, 19, 127, 3187	R9 33 994662872 2E3, 11, 19, 113, 659, 883
R9 33	7939179525 2.5, 11, 19, 127, 3187	L1 43 1179315 2E2, 5.1, 31, 79, 107
1295	79615350 2.5E2, 7.17, 179, 121439	L1 43 12121076 2E2, 31, 47, 2267
R9 33	805591450 2.5E2, 7.17, 179, 121439	L1 43 14242230 2.7, 5.13, 83, 191
1308	8071769104 2E5, 11, 19, 223, 557	L1 43 1668988 2.7, 31, 71, 587
R9 33	83363880965 2E5, 11, 19, 223, 6079	L1 43 18299715 2E2, 5.1, 11, 59, 89
1312	792509717 2E4, 7, 11, 13, 19, 307	L1 43 1844267930 2.5, 11, 197, 571, 1259
R9 33	8013961683 2E4, 7, 11, 13, 19, 307	R9 33 992985128 2E3, 11, 19, 103, 3229
1314	794148664 2E3, 9, 17, 193, 319	R9 33 8013961683 2E4, 7, 11, 13, 19, 307
R9 33	8013961683 2E4, 7, 11, 13, 19, 307	L1 43 1179315 2E2, 5.1, 31, 79, 107
1327	820184795 3.5E2, 7.17, 179, 121439	L1 43 12121076 2E2, 31, 47, 2267
R9 33	8433856485 3.5E2, 7.17, 47, 6, 3869	L1 43 14242230 2.7, 5.13, 83, 191
1330	829484116 2E2, 1, 19, 89, 116077	L1 43 1668988 2.7, 31, 71, 587
R9 33	8515214884 2E2, 1, 19, 89, 116077	L1 43 18299715 2E2, 5.1, 11, 59, 89
1336	8283367504 2E4, 7, 11, 13, 19, 307	L1 43 1844267930 2.5, 11, 197, 571, 1259
R9 33	88033961683 2E4, 7, 11, 13, 19, 307	R9 33 88033961683 2E4, 7, 11, 13, 19, 307
1340	834838050 2.5E2, 11, 319, 479	R9 43 1668988 2.7, 31, 71, 587
L1 33	8603384850 2.5E2, 17, 229, 4519	R9 43 8788952 2.5, 11, 19, 83, 16631
1343	845251584 2E5, 59, 151, 2963	R9 43 8788952 2.5, 11, 19, 83, 16631
R9 33	848775456 2E5, 59, 151, 2963	R9 43 89169695 3.5E2, 11, 19, 83, 16631
1352	851706125 3E3, 5E3, 11, 83, 559	R9 43 934242572 2E3, 31, 383, 1979
R9 33	873245875 3E3, 5E3, 11, 83, 559	R9 33 873245875 3E3, 5E3, 11, 83, 559

Appendix III

The gcd's of the first 1427 APs

GCD	FREQ	RANK	NUMBER(S) OF AP'S WITH THIS GCD									
2	2	2	278									
4	67		1	3	4	23	24	36	53	64	83	97
			115	153	154	165	209	226	238	255	267	274
			294	313	323	345	347	348	502	505	578	586
			632	645	721	738	748	750	763	776	808	887
			811	822	842	846	852	867	877	909	928	937
			975	976	1060	1119	1132	1160	1251	1307	1316	1322
			1350	1355	1365	1368	1374	1399	1406			
8	208		5	6	17	21	26	31	32	33	34	35
			37	49	42	47	59	62	65	67	69	74
			76	80	81	84	90	93	94	95	103	110
			112	116	120	122	126	131	140	143	144	148
			156	159	161	163	174	180	182	184	186	192
			199	201	206	208	212	217	218	239	240	244
			258	266	279	280	293	296	298	300	305	311
			314	315	317	318	330	333	339	353	357	361
			363	374	375	376	382	387	400	405	417	432
			438	447	452	455	458	459	463	467	475	498
			513	523	525	546	552	553	558	571	574	579
			588	587	591	599	606	631	633	639	644	646
			652	658	665	668	690	702	713	716	720	722
			727	737	762	771	788	806	808	814	836	845
			848	854	855	858	863	869	871	876	881	912
			913	919	925	942	946	953	972	973	974	978
			981	982	992	999	1011	1026	1028	1043	1063	1077
			1078	1079	1113	1117	1120	1126	1130	1141	1152	1153
			1164	1170	1182	1184	1213	1218	1222	1235	1243	1250
			1252	1253	1279	1302	1320	1329	1345	1354	1356	1361
			1376	1378	1391	1393	1404	1407	1408	1417		
10	94		13	18	43	45	49	52	54	55	58	70
			75	79	98	99	105	107	128	146	190	218
			219	227	228	264	275	283	286	289	299	328
			332	343	378	420	437	441	448	476	479	507
			519	534	544	589	598	609	617	620	627	664
			670	674	676	723	759	769	826	834	835	843
			859	897	904	1001	1007	1023	1027	1042	1053	1062
			1864	1066	1074	1093	1103	1104	1105	1108	1111	1123
			1148	1159	1167	1209	1221	1240	1257	1266	1285	1287
			1373	1394	1398	1419						
14	30		25	61	73	77	119	127	135	177	187	198
			249	377	395	399	412	470	480	651	701	756
			790	809	825	857	873	1070	1110	1342	1344	1384
15	1		900									

6	6	150	8	16	16	19	20	27	29	51	60	76	11	207	269	543	565	569	816	874	1071	1255	1272				
6.3	6.8	86	118	121	156	164	169	175	175	175	175	92	4	1288	200	515	612	612	816	874	1071	1255	1272				
1.79	1.85	193	196	213	232	249	257	261	261	261	261	98	6	200	515	612	612	612	816	874	1071	1255	1272				
2.84	2.85	288	307	326	322	328	327	336	336	336	336	105	24	28	273	585	711	1089	1165	816	874	1071	1255	1272			
3.65	3.66	385	396	407	429	433	434	454	454	454	454	105	24	28	273	585	711	1089	1165	816	874	1071	1255	1272			
4.35	4.45	450	453	454	474	486	497	509	509	509	509	105	24	28	273	585	711	1089	1165	816	874	1071	1255	1272			
5.27	5.36	545	545	545	573	575	578	583	583	583	583	105	24	28	273	585	711	1089	1165	816	874	1071	1255	1272			
6.43	6.54	669	671	672	673	682	692	696	696	696	696	105	24	28	273	585	711	1089	1165	816	874	1071	1255	1272			
7.86	7.93	742	744	778	799	828	846	866	866	866	866	105	24	28	273	585	711	1089	1165	816	874	1071	1255	1272			
8.94	9.03	906	918	930	945	958	962	966	966	966	966	105	24	28	273	585	711	1089	1165	816	874	1071	1255	1272			
9.70	10.06	1010	1016	1024	1035	1035	1041	1041	1041	1041	1041	110	20	109	292	349	494	623	661	802	922	922	105	1255	1272		
10.75	10.86	11.06	11.16	11.25	11.27	11.28	11.29	11.29	11.29	11.29	11.29	110	20	109	292	349	494	623	661	802	922	922	105	1255	1272		
11.39	11.46	11.54	11.60	11.67	11.92	11.94	11.98	12.02	12.02	12.02	12.02	110	20	109	292	349	494	623	661	802	922	922	105	1255	1272		
12.14	12.27	12.38	12.41	12.56	12.61	12.68	12.74	12.74	12.74	12.74	12.74	120	20	109	292	349	494	623	661	802	922	922	105	1255	1272		
12.91	12.97	13.26	13.28	13.36	13.44	13.67	14.11	14.11	14.11	14.11	14.11	120	20	109	292	349	494	623	661	802	922	922	105	1255	1272		
1	1297	1326	1328	1336	1344	1367	1411	1411	1411	1411	1411	120	20	109	292	349	494	623	661	802	922	922	105	1255	1272		
316	316	316	316	316	316	316	316	316	316	316	316	128	2	442	983	128	128	128	128	128	128	128	128	128	128		
76	76	89	89	89	1003	1003	1003	1003	1003	1003	1003	130	16	104	718	104	104	104	104	104	104	104	104	104	104	104	
287	324	335	344	346	352	371	388	411	413	413	413	130	16	104	718	104	104	104	104	104	104	104	104	104	104	104	
419	456	468	471	471	525	525	540	548	548	548	548	130	16	104	718	104	104	104	104	104	104	104	104	104	104	104	
595	616	655	686	710	757	761	827	856	856	856	856	130	16	104	718	104	104	104	104	104	104	104	104	104	104	104	
860	876	913	923	923	957	957	960	979	979	979	979	130	16	104	718	104	104	104	104	104	104	104	104	104	104	104	
991	1003	1058	1067	1072	1118	1122	1163	1166	1178	1178	1178	130	16	104	718	104	104	104	104	104	104	104	104	104	104	104	
1210	1223	1223	1246	1246	1308	1310	1341	1343	1343	1343	1343	130	16	104	718	104	104	104	104	104	104	104	104	104	104	104	
1382	1383	1388	1395	1396	1409	1409	1409	1409	1409	1409	1409	130	16	104	718	104	104	104	104	104	104	104	104	104	104	104	
44	44	44	44	44	44	44	44	44	44	44	44	144	2	444	987	144	144	144	144	144	144	144	144	144	144	144	144
596	683	688	692	692	911	914	915	932	944	944	944	944	152	16	125	366	436	559	600	805	812	831	879	895	895	895	
829	868	872	872	872	872	872	872	872	872	872	872	152	16	125	366	436	559	600	805	812	831	879	895	895	895		
821	837	864	864	864	864	864	864	864	864	864	864	152	16	125	366	436	559	600	805	812	831	879	895	895	895		
1038	1096	1107	1113	1113	1190	1190	1284	1287	1287	1287	1287	152	16	125	366	436	559	600	805	812	831	879	895	895	895		
1290	1346	1406	1406	1406	1406	1406	1406	1406	1406	1406	1406	152	16	125	366	436	559	600	805	812	831	879	895	895	895		
52	13	133	133	133	135	135	135	135	135	135	135	150	16	125	366	436	559	600	805	812	831	879	895	895	895		
44	71	124	129	181	223	263	272	356	356	356	356	150	16	125	366	436	559	600	805	812	831	879	895	895	895		
404	423	451	500	512	551	739	751	797	797	797	797	150	16	125	366	436	559	600	805	812	831	879	895	895	895		
891	902	987	1015	1076	1135	1284	1287	1289	1289	1289	1289	150	16	125	366	436	559	600	805	812	831	879	895	895	895		
68	3	362	402	402	402	402	402	402	402	402	402	150	16	125	366	436	559	600	805	812	831	879	895	895	895		

SUPPLEMENT

236	1	1301		530	1	1386
238	2	1414		548	1	1304
248	4	276	392	574	1	1647
250	1	588		585	41	15
255	1	705		592	1	478
256	4	520	556	632	1	1018
266	8	308	368	664	1	564
273	1	78		678	1	243
285	2	123	968	675	15	147
286	1	764		682	1	1168
298	2	191	1422	688	3	369
296	3	151	183	692	1	1031
310	11	66	204	693	1	1081
315	13	1323	738	703	1	944
322	1	145	1197	703	5	754
328	1	936		703	15	642
376	1	557		703	19	642
405	5	259	379	703	19	841
424	2	747	1234	703	19	889
434	5	463	496	703	19	1375
465	1	622		703	19	1375
484	2	712	884	703	19	1375
488	2	576	890	703	19	1375
495	4	516	533	703	19	1375
512	1	1239		703	19	1375
518	1	517		703	19	1375
525	17	410	521	703	19	1375
1158	1173	1175	1225	703	19	1375

908	1	988		2132	1	685
950	3	563	1142	2154	1	838
975	1	448		2152	1	
1016	2	875	1138	2295	2	1338
1028	1	1427		2415	2	615
1035	1	954		2457	1	492
1048	1	446		2528	1	1265
1072	1	233		2541	1	1183
1078	2	370	636	2565	3	113
1084	1	765		2625	2	430
1150	3	230	648	731		660
1155	3	528	729	1397		1019
1210	1	1149		2895	1	1282
1215	1	1054		28112	1	1369
1276	1	1065		2925	9	1313
1365	3	541	840	1137		253
1395	3	186	547	621		803
1425	2	469	1392			819
1485	5	149	250	337		933
1550	13	111	241	383		941
1781	1	929	1055	1339		1049
1795	8	245	306	356		1080
1862	2	1231		443		1248
1911	1	134	1249	542		
1972	1	685		782		
1995	4	1377		1133		
2025	1	522	529	955		
2079	2	401		1179		
2096	1	926	952			
		341				

1	5296	1	583	2	488	766	501	532	908	928	1039	1046	1109	18135	2	893	1284	
1	5335	2	488	766	501	532	908	928	1039	1046	1109	1128	1263	18837	1	678	1228	
1	5733	16	46	278	312	501	532	908	928	1039	1046	1109	20925	6	717	1228		
1	6227	1	1296	6622	1	1121	6885	3	386	791	1327	6975	1	1088	21068	1	918	1263
1	7038	1	1271	7564	1	367	7685	1	524	8775	2	425	940	9089	3	268	695	743
1	9298	1	1089	9495	1	1418	9975	2	1057	1291	111115	2	297	495	11475	1	689	11475
1	13041	1	785	13336	1	581	13923	1	1232	13965	2	1196	1387	14553	1	768	14553	
1	14445	1	741	14535	1	1224	14535	1	1224	14535	1	1196	1387	15561	2	1254	1312	
1	15795	1	1358	16245	1	364	16275	1	256	16275	1	1196	1387	15795	1	1358	16245	